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Intergenerational Risk Trading and the Innovative Role of Equity-Wage Swaps¹

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¹ The views expressed here are those of the authors and not necessarily those of any affiliated institution.

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Abstract

From a life cycle theory perspective, both young and old individuals may gain from a reallocation of equity and wage risk exposure between each other. However, current financial markets do not offer wage growth-linked securities and borrowing against labor income without collateral is difficult. To improve intergenerational risk reallocation, we propose a market-based voluntary risk trading arrangement between coexisting generations via an innovative swap market where participants trade equity-related returns for wage-linked returns, and vice versa. The maturity of the swap contract is restricted to one year to address the collateral issue. We find there is always a market for equity–wage swaps and the market-clearing premium will vary depending on multiple state variables (economy, demographics, and human and financial capital). This innovative swap market is effective at improving the welfare of all generations because the trading of wage-linked returns leads to a more complete market, enabling individuals to realize a more preferred risk exposure over their life cycles.

JEL classification: D52, G11, G23

Keywords: Intergenerational risk trading, equity–wage swap market, wage-linked claims, incomplete market, equilibrium pricing

1. Introduction

Participants of defined-contribution (DC) plans increasingly invest their pension wealth in a default asset mix typically composed by professionals from the pension industry. These default strategies increasingly reflect the insights of the life cycle investment approach (Viceira 2008), where equity exposure typically declines with age to be replaced by less risky or even risk-free assets. Given the uncertainties in one's labor income (and hence one's human capital), optimal life cycle asset-allocation strategies would preferably include wage-linked claims. This strategy cannot be implemented as a default mix, however, because of the lack of wage-linked financial assets. This study proposes the innovation of a market-based voluntary risk-trading vehicle between coexisting generations for trading equity risk and wage risk, potentially enhancing welfare.

Wage-linked benefits have typically been available in funded defined-benefit (DB) pension plans, but in the last decades these plans have been on the retreat worldwide. For this plan type, accrued pension rights are typically wage-indexed and, after retirement, benefits could be wage-indexed. In many plans, funding risks are borne collectively by younger and older participants. The allocation of risk and return over plan members is defined by the content of the pension deal. This type of intergenerational risk sharing is potentially welfare improving for various reasons (Gollier 2008; Cui *et al.* 2011), particularly because of the implicit offering of wage-linked claims. A main drawback of this framework is that mandatory risk sharing between generations may not be maintainable under all circumstances, due to the size of the implied redistributive transfers between generations.

Recognizing the potential welfare enhancement of intergenerational risk sharing via pension schemes, this study investigates the design of a market-based voluntary risk-trading vehicle between coexisting generations. Specifically, we look at the creation of a so-called equity–wage swap (EWS) market, where individual pension plan participants voluntarily trade equity exposure and wage exposure at market-clearing prices. We identify the natural counterparties in such a market, based on the insights from life cycle theory. Young individuals with relatively a too-large fraction of personal wealth as human capital might be interested in adding more equity exposure to their pension wealth and providing wage-linked exposure in exchange, whereas older individuals with a relatively low fraction of wealth in human capital might wish to add wage-linked returns to their pension wealth and reduce their financial risk exposure. In the current financial market, their wishes cannot be implemented because one (especially the young) cannot borrow against their human capital without collateral and because the market is incomplete because it does not offer tradable wage-linked securities. However, individuals may find that engaging in EWS trading helps them to achieve their optimal exposure in a flexible manner.

We set the maturity of the swap to one year. Every year, individuals can choose how much equity risk and wage risk to swap. We solve a life cycle model with multiple cohorts to characterize the optimal exposure of all co-existing generations and to find the equilibrium price for the swap. An important driver behind the trading and pricing of risk is the ratio of human capital to financial capital. This ratio is state-dependent and therefore may not be in line with the preferred size. By swapping human and financial capital, individuals can restore this ratio toward the preferred size.

This paper builds on and contributes to three fields of literature: on optimal asset allocation over the life cycle (Bodie *et al.* 1994; Cocco *et al.* 2005; Cui 2009), on innovative securities for pension planning and retirement savings (such as longevity-linked securities, Blake and Biffins 2012; wage-linked bonds, de Jong 2008a, 2008b; and real annuities, Brown *et al.* 2001 and van Binsbergen *et al.* 2013) and, finally, on substitutes for intergenerational risk-sharing arrangements (Beetsma and Buccioli 2011; Molenaar and Ponds 2012; van Binsbergen *et al.* 2013).

In a stylized setting without interim consumption and a labor income stream, de Jong (2008b) derives the equilibrium price of wage-linked bonds for an investor optimizing utility on wage-deflated wealth. It is shown that investors with relatively low risk aversion and/or a high ratio of human capital to total wealth become the issuers (or net suppliers) of wage-linked bonds. Investors with relatively high risk aversion and/or a low ratio of human capital to total wealth become the holders (or net demanders) of these wage-linked bonds. Furthermore, the equilibrium price of wage risk in de Jong's study is determined from the condition that the aggregate demand for wage-linked bonds has to be zero.

In a setting with interim consumption and a labor income stream, however, the optimal demand for wage-linked assets will be affected. This paper aims to shed light on the demand and supply of the EWS in a realistic setting, including a stochastic labor income stream, social security, income tax, and housing and medical expenditures for multiple cohorts. We develop a dynamic equilibrium model with multiple cohorts to investigate the evolution of the market size, the market making process, and the market-clearing (equilibrium) premium.

In our theoretical framework, we find that there is always a market for trading EWSs among co-existing generations and that the market-clearing premium from this risk trading may vary, depending on multiple state variables (economy, demographics, and human and financial capital). Since the maturity of the swap contract is restricted to one year, the collateral issue is easily met in practice. The proposed EWS market can be established because 1) EWSs help to complete a missing market of wage-linked claims, 2) EWSs help to lift the liquidity (borrowing) constraints of the young, and 3) EWSs help individuals by offering a flexible way to realize their preferred exposure to wage risk and equity risk.

The EWS product is flexible regarding who can participate. Natural candidates for organizing the EWS market would be the traditional parties involved in assisting individuals in their retirement income planning, such as pension funds—which would then function as intermediary and clearing house.

DC plans in many countries are currently growing in size and coverage but are still relatively small, compared to DB plans. Many people, especially older generations, are still primarily covered by DB plans. This paper therefore serves as a thought experiment for a possible future situation in which DC plans become the dominant form of pension plan.

The rest of the paper is organized as follows. Section 2 describes the economy, the preference structure of individuals and the labor income process. After introducing the EWS instrument, we present the individuals' optimal consumption model and the optimal EWS choice framework. Section 3 discusses the results in terms of how the market size, the market-clearing (equilibrium) premium and the optimal choices of the multiple cohorts over their life cycle evolve. We present the baseline case and four specific cases. Analysis of these cases demonstrates the flexibility of the EWS construction, which is able to adapt to any given default setting. Section 4 concludes the paper.

2. Model setup

2.1 Generations and their labor income processes

The economy is populated with investors from different age cohorts, from young (workers) to old (retirees), and each cohort is populated with a number of non-identical individuals. We assume that all individuals start working at age 25 and retire at 65 and that the maximal obtainable age is 95. The model thus consists of 70 age cohorts coexisting at each point in time. In the baseline setup of the paper, we assume a stable population. All cohorts are of equal size when entering the labor market at age 25. The size of each cohort shrinks over time according to the assumed survival probabilities. In the baseline

result, the survival probability is assumed to be driven by a constant mortality force. Section 3.3 uses the projected survival probabilities of the United Nations.

During the working period, individuals earn a stochastic labor income stream, denoted Y . Let t denote the calendar year and t_x the birth year of individual x , where $t - t_x$ is the age ($25 \leq t - t_x \leq 95$) of individual x . Following Benzoni et al. (2007), we assume that individuals' real labor income $Y_{t,t_x,x} = G_t N_{t,t_x,x}$ can be decomposed into two components: a stochastic aggregate wage component G_t and a stochastic age-dependent idiosyncratic component $N_{t,t_x,x}$. The growth rate of the aggregate wage component is stochastic, determined according to equation (2)⁴ below, where \bar{g} and σ_g denote the mean and volatility of the annual aggregate wage growth. The idiosyncratic wage component, N , has an age-dependent drift $f(t - t_x) = a_0 + a_1(t - t_x)$ that generates the hump shape of earnings and log-normally distributed permanent shocks. The idiosyncratic wage rate shocks $\eta_{t,t_x,x}$ follow the standard normal distribution, which generates idiosyncratic permanent income shocks. The real labor income process is thus specified as follows:

$$Y_{t,t_x,x} = G_t N_{t,t_x,x} \quad (1)$$

$$R_t^G = \ln(G_t/G_{t-1}) = \bar{g} + \sigma_g \epsilon_{g,t} \quad (2)$$

$$\ln N_{t,t_x,x} = \ln N_{t-1,t_x,x} + f(t - t_x) + \sigma_n \eta_{t,t_x,x} \quad (3)$$

In the baseline model, we assume $\bar{g} = 0.8\%$ and $\sigma_g = 4\%$ per year. The starting annual salary at age 25, Y_{25} , is normalized to \$20,000. The parameters a_0 and a_1 are set according to the calibration of Cocco et al. (2005) for the high-education group ($a_0 = \$0.066$ and $a_1 = -0.0024$)⁵ and $\sigma_n = 8\%$, assumptions that are also adopted in the literature (e.g., Benzoni et al. 2007).

After retirement, individuals receive a social security benefit financed on a pay-as-you-go basis. Social security benefits at age 65, S , are assumed to be a fraction s of the final labor income, that is, $S = s * Y_{65,x}$. Each year, social security is indexed with the aggregate wage growth. In the baseline model, we consider $s = 30\%$. The social security contribution rate (about 20%) is included in the income tax rate for employees.

During the retirement period, individuals consume their accumulated wealth, denoted $W_{t,x}$, and social security benefits. Upon death, any remaining wealth in the DC plan is paid out to the individual's beneficiary, since there is little consensus in the literature on the strength of pure bequest motives; furthermore, we assume in this paper that all bequests are accidental.

2.2 Financial markets

Two financial assets are traded in the financial markets, one risk-free and one risky (both in real terms). The real risk-free asset offers a fixed real interest rate r . The real price of the risky stock index, E_t , follows a geometric Brownian motion with constant drift. Dividends are reinvested. The uncertain stock

⁴ Aggregate wage growth and equity returns are cointegrated by Benzoni et al. (2007); asset allocation to equities may thus be reduced due to the cointegration effect. Given this paper's focus on the trading and pricing of EWSs with a one-year horizon, we assume that aggregate wage rates and equity returns are correlated but not cointegrated.

⁵ Our main results for the EWS premium remain very similar for other income profiles. Therefore, only the results based on the high-education group are shown in this paper.

returns are potentially correlated with stochastic aggregate wage growth. The stock return and wage growth-rate dynamics are as follows:

$$R_t^E = \ln(E_t/E_{t-1}) = \mu + \sigma_E \epsilon_{E,t}$$

where μ and σ_E are the mean and volatility of annual stock returns. In the baseline model, we assume $r = 2\%$, $\mu = 5\%$, and $\sigma_E = 15\%$, annualized.⁶ Furthermore, we set the correlation between stock return growth R_t^E and aggregated wage growth (R_t^G in equation (2)) at 0.2.

2.3 The equity-wage swap

In the current financial market, standalone wage-linked securities are not available. Wage inflation-indexed social security and traditional DB pensions can be interpreted as implicit wage-linked assets, but they are not tradable and tend to disappear due to the ongoing shift from DB to DC pension schemes. This paper examines the potential innovation of tradable wage-linked securities, namely, EWSs.

The maturity of the swap is one year, which provides maximal flexibility to adapt a preferred position in the swap on a yearly base.⁷ Every year, individuals can choose how much equity risk and wage risk to swap. Suppose L is the principal or the face amount of the swap. The swap agreement is reached at time t between two counterparties, with party A willing to supply wage exposure R_{t+1}^G and to demand equity exposure at R_{t+1}^E from party B. Figure 1 displays the cash flows involved in the swap. The initial capital outlay is zero by construction. At time $t + 1$, the payoff for party B (the wage leg receiver) is $L(1 + R_{t+1}^G)$. For party A (the equity leg receiver), the payoff can be rewritten as $L(1 + R_{t+1}^E - m_t)$, where m_t is the market-clearing premium determined by the total supply and demand of such EWSs by all participants. The dynamics of this premium are discussed in more detail in Sections 2.5 and 2.6. The net cash flow for party A at time $t + 1$ is therefore $L(R_{t+1}^E - R_{t+1}^G - m_t)$.

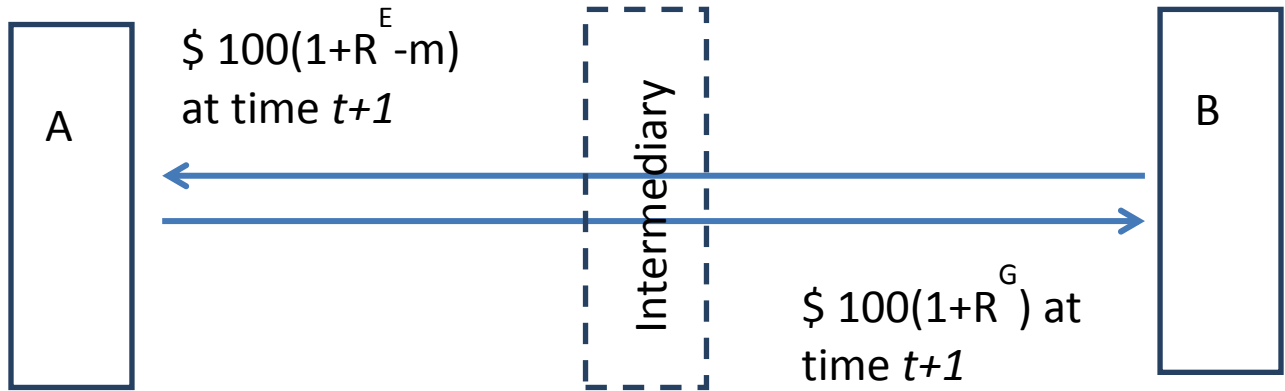


Figure 1: Indirect equity-wage swap (illustrative).

EWSs may have several potential advantages. First, they can help individuals to realize their preferred allocation of risk exposure over a lifetime. The old can acquire wage growth exposure without a labor income contract. The young can obtain equity exposure without physically holding stocks. Since the swap requires zero initial capital from the swap-holders, the swap lifts part of the borrowing constraint of

⁶ Given these parameter assumptions, a myopic investor with a risk aversion of three, would optimally allocate 44% in equities and 56% in risk-free assets.

⁷ Analysis of the potential market for longer maturity swaps is one of our future research topics.

younger workers. Second, EWSs are traded among parties based on voluntary participation, with risk exposure traded at transparent market fair prices. These features of the equity swap market can be seen as an improvement to a traditional DB plan, with its implicit premiums and mandatory sharing of risks among plan participants.

2.4 Investor preferences

Individuals derive utility over consumption, denoted C_{t,t_x} . The preferences of each individual are captured by the constant relative risk-aversion utility function

$$E \left[\sum_{t-t_x=25}^{95} \beta^{t-1} p_{t-t_x} \frac{C_{t,t_x}^{1-\gamma}}{1-\gamma} \right],$$

where γ is the risk-aversion parameter, β is the subjective discount factor, and p_{t-t_x} is the survival probability from age 25 up to age $t - t_x$. In the baseline case, we opt for $\gamma = 3$ and $\beta = 0.97$, following the classical life cycle literature (e.g., Cocco et al. 2005).

2.5 Investors' optimal choice problem

Throughout their life cycle, individuals make optimal choices regarding their consumption and hence savings. During the working period, all individuals optimally save a portion of labor income to finance their retirement consumption and then consume the rest of their labor income. We further assume that individuals save for retirement consumption via a DC pension plan with a default asset mix⁸ in which risky assets comprise a fraction α^{DC} and risk-free assets assume a fraction $(1 - \alpha^{DC})$. The default equity exposure can be either constant or age-dependent ($\alpha_{t-t_x}^{DC}$). The EWS enlarges the investment universe with wage-growth risk exposure. In this paper we assume that individuals do not change the default asset mix in the DC plan and only change risk exposure to equity and wage growth by taking EWS positions.

Formally, individuals optimize expected utility by choosing optimal consumption C_{t,t_x} and the optimal position K_{t,t_x} in the EWS each year. A positive position ($K > 0$) means adding (long) equity exposure and reducing (short) wage exposure, whereas a negative position ($K < 0$) implies reducing equity exposure and adding wage exposure. Naturally, a zero position ($K = 0$) means not participating in the swap market.

The market-clearing condition pins down the fair premium m . Let the variable $K_{t,t_x}[m_t]$ denote individual x 's choice of the optimal notional amount of EWS at time t , for a given premium m_t . The optimization problem of each individual is as follows:

$$\max_{\{C_{t,t_x}, K_{t,t_x}[m_t]\}} E \left[\sum_{t-t_x=25}^{95} \beta^{t-1} p_{t-t_x} \frac{C_{t,t_x}^{1-\gamma}}{1-\gamma} \right]$$

The budget constraint of the individual, with W_t standing for wealth at the end of each period, is

$$W_{t+1,t_x,x} = (W_{t,t_x,x} - C_{t,t_x,x}) (\alpha^{DC} R_{t+1}^E + (1 - \alpha^{DC}) R_t^f) + K_{t,t_x} (R_{t+1}^E - R_{t+1}^G - m_t) + Y_{t+1,t_x}$$

⁸ We assume that individuals do not have other sources of wealth apart from their pension savings.

We impose $0 \leq \alpha^{DC} \leq 1$ in order to address borrowing and short-selling constraints. Furthermore, we impose liquidity constraints, so that cash on-hand must be non-negative, $W_{t,t_x} - C_{t,t_x} \geq 0$, and the EWS notional cannot exceed cash on-hand, $|K_{t,t_x}| < W_{t,t_x} - C_{t,t_x}$.

2.6 Market-clearing and equilibrium pricing

To pin down the equilibrium premium m , the process of market clearing and price setting is executed in two steps. First, individuals reveal their preferred supply and demand for a range of possible premiums. The information is then aggregated to set the premium at the market-clearing level of the EWS premium m_t^* so that total supply equals total demand:

$$\sum_x K_{t,t_x}(m_t^*) = 0$$

The demographic structure thus has an impact on the market-clearing outcome. In sections 3.1-3.3 we assume a static demographic structure, and in section 3.4 we apply a changing demographic structure because of aging.

2.7 Solution method

We generate a set of 6000 scenarios based on the assumptions defined in sections 2.1 and 2.2. Then we implement the dynamic programming method using simulations. To do so, we define two new variables, investable wealth, A_{t,t_x} , with $A_{t,t_x} = W_{t,t_x} - C_{t,t_x}$, and α_{t,t_x}^K , the portfolio weight of EWS relative to investable wealth, with $\alpha_{t,t_x}^K = K_{t,t_x,x}/A_{t,t_x,x}$. We can rewrite the budget constraint as

$$W_{t+1,t_x} = A_{t,t_x}[(\alpha^{DC} R_{t+1}^E + (1 - \alpha) R_t^f) + \alpha_{t,t_x}^K (R_{t+1}^E - R_{t+1}^G - m_t)] + Y_{t+1,t_x}$$

We build on the dynamic programming technique (developed by Carroll 2006, and extended by Cui 2009) to solve the optimal consumption policy and the investment policy $K_{t,t_x}[m_t]$. That is, individuals' supply and demand are determined by first-order conditions with regard to consumption and notional swap size for a given wealth level and any given EWS premium at any time t . The first-order condition is expressed in the form of a conditional expectation that includes not only future equity and wage growth uncertainties, but also future EWS premiums. Since future EWS premiums are not known at time t , investor decisions are based on prior distributions of future EWS premiums. This solution technique is inspired by the method proposed by Gomes and Michaelides (2008) for computing equilibrium factor returns. The details of the solution steps are discussed in the Appendix.

By using this dynamic programming method, we ensure that individuals' choices in each step are optimal and that undesirable income states are avoided endogenously. In fact, the dynamic programming technique operationalizes the principle of "begin with the end in mind," which is a very good principle for retirement planning. The technique of defining retirement saving objectives and working backward to achieve them with high probability is strongly recommended by several scholars (cf. Merton 2010; Blake et al. 2008).

3. Results

There are three key questions that we want to investigate. First, will the market for EWS exist, so that some individuals indeed have an incentive to take the equity-linked leg and other individuals, as the counterpart, have an incentive to take the wage-linked leg, such that the market clears? Second, how large could the market be, in terms of open interest? Finally, what level of premium is required in equilibrium?

We find that these quantities are driven by four important factors: first, the preferences of the individual over the life cycle and the assumed economy, both of which were specified in the previous section; second, the path-dependent history of the individual as reflected by the ratio of human capital to financial capital; third, the default asset mix provided by the pension fund, either a constant mix or a life cycle mix; and, fourth, the evolution of the demographic structure, either stable or aging. Obviously, these factors will affect the EWS positions that individuals choose. The construction of an EWS is so flexible that it can be adapted to any given default setting or demographic structure.

This section first addresses the question of whether the EWS market comes alive at all. We find a positive answer for the baseline case, and then evaluate alternative settings. In particular, we address the impact of alternatives for the default asset mix, for the risk preference and the demographic structure, and for a higher correlation between equity and wage growth rates.

3.1 Would the EWS market come alive and why?

We evaluate the demand, supply and resulting market-clearing prices of EWSs for a default asset mix consisting of 44% risky assets and 56% risk-free assets. The population is assumed to be stable, so that the demographic structure (i.e. cohort sizes) remains constant over time. All cohorts are of equal size when entering the labor market at age 25. The size of the cohorts shrinks over time according to survival probabilities. Section 3.4 discusses the impact of changing demographic structure due to the longevity trend on the demand of EWSs.

Let us imagine that the swap vehicle is available at a certain price. Do individuals have an incentive to take a position, either long or short, in this market? Figure 2 depicts the optimal EWS choice at different ages, expressed in terms of $\alpha_{t,t_x,x}^K = K_{t,t_x,x}/A_{t,t_x,x}$ when the premium m varies from -100 basis points (bps) to 300 bps (see the legend of Figure 2). The four panels show the optimal choice for four different ages (30 years, 50 years, 65 years and 85 years). Each panel lists the level of financial wealth in units of labor income along the horizontal axis. The vertical axis shows the optimal weight of the swap in the individuals' portfolios. A positive weight means that the individual would like to increase equity exposure in exchange for wage growth exposure. The panels indicate that younger individuals overall prefer a positive position in EWSs, whereas the elderly tend to opt for a negative position, indicating that they prefer to add wage growth exposure to their financial capital.

We recall that an EWS is an exchange between future wage growth R_{t+1}^G and future equity return minus the premium $R_{t+1}^E - m_t$; the payoff of the swap is thus $R_{t+1}^E - R_{t+1}^W - m_t$. Note that the higher the premium, the less attractive it is to demand extra equity exposure (and hence the lower the preferred position in EWS).

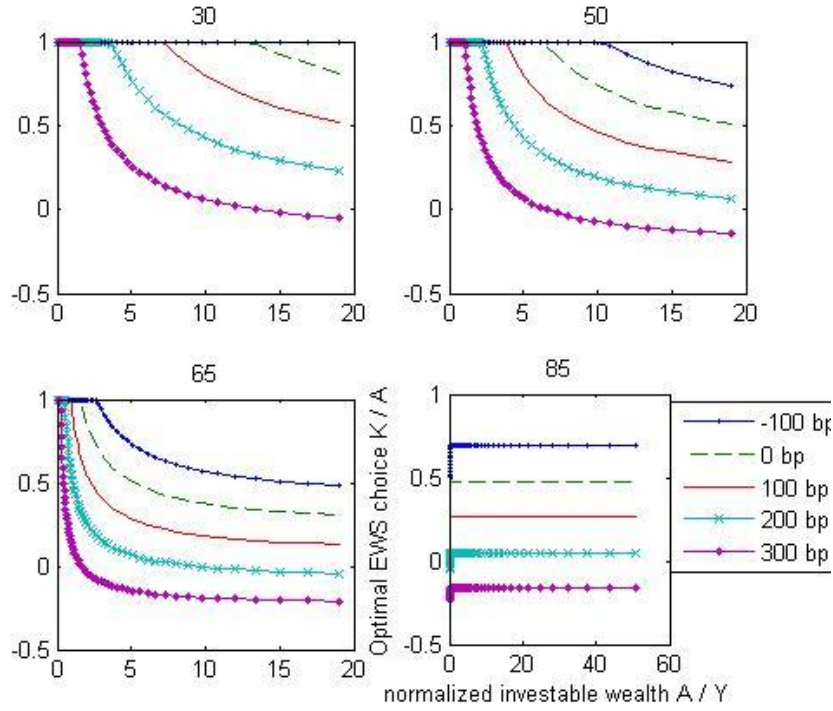


Figure 2: The optimal policy function of EWS weights for four specific age cohorts for different premium levels (m) and different levels of investable wealth (A) in a setting with a constant default asset mix of the DC plan. The x-axis shows investable wealth normalized to labor income (A/Y); the y-axis shows the optimal portfolio weight in the EWS ($\alpha_{t,t,x}^K$).

Now we return to our key questions: Will the market find an equilibrium, and for what level of the premium m ? Figures 3 and 4 show the evolution of the premium level and the EWS market's size over the entire simulated 70-year horizon displayed along the x-axis. When the EWS market starts up, all cohorts initially hold their capital in the same default asset mix, consisting of the risk-free assets and the risky investment. They all will experience a mismatch with their preferred asset mix, which will vary with age.

Figure 3 displays the movement in the market-clearing price over time. The initial value of the premium is larger than its longer run value. This suggests that the initial mismatch of the default mix with the preferred one is relatively larger for the younger cohorts than for the older ones. The young are willing to pay a large m in order to get more equity risk exposure. The expected return of the long position in the swap is in any period t equal to: $E[R^E] - (E[R^G] + m_t)$. A higher value of m will lead to a lower but still-positive expected return on a long position. The elderly choose to have a short position in the swap, as they are willing to accept low-yielding wage growth exposure in exchange for high-risk equity exposure. After the initialization of the swap market, a process of capital transfers from older to younger cohorts will take place. As time elapses, more and more workers have been able to acquire additional capital via their swap positions, and the additional financial capital helps them to get a higher equity risk exposure vis-à-vis the remaining human capital. It takes time before the aggregate human capital risk exposure and the aggregate equity exposure are adjusted toward a more optimal situation by cohorts who have had the opportunity to participate in the EWS market after entering the labor market. Actually, the adjustment period encompasses 40 years, after which all working cohorts have had the opportunity to

participate in the EWS market. Initial economic conditions will have an impact on the specific path by which the economy finds the new equilibrium.

Figure 4 indicates the market size relative to total wealth accumulated in the economy of all coexisting cohorts. We see that the relative market size initially amounts to 10–12% of the accumulated capital, but gradually stabilizes at 5–8%. This informs us about the gradual evolvement of the size of the swap market. Initially, the financial capital only consists of investments in the risk-free asset and the risky equity. After the introduction of the EWS instrument, capital is redistributed from the old to the young, and this shift of capital enables the full span of cohorts to realize a more preferred exposure to risks. The EWS market helps the elderly to have exposure to wage growth; in turn, they accept a lower capital return. The young get a higher equity risk exposure and a larger financial capital growth.

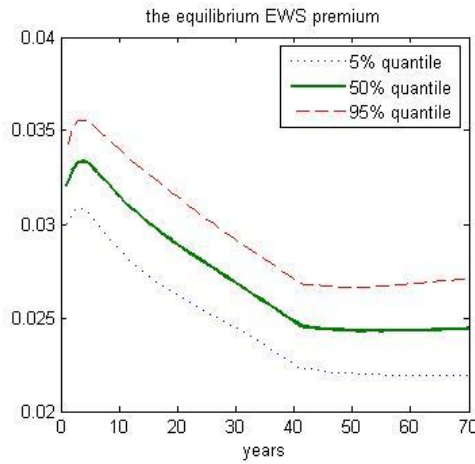


Figure 3: The 5%-, 50%- and 95% quantiles of the market-clearing premium (in %) over the simulated 70-year horizon in a setting with a constant default asset mix in the DC plan.

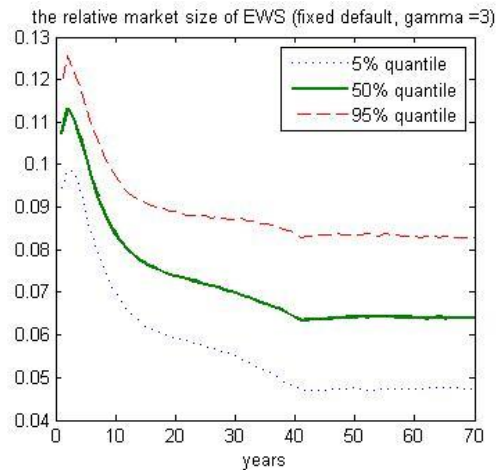


Figure 4: The 5%, 50% and 95% EWS market size quantiles relative to the total accumulated wealth of all coexisting cohorts over time, in a setting with a constant default asset mix.

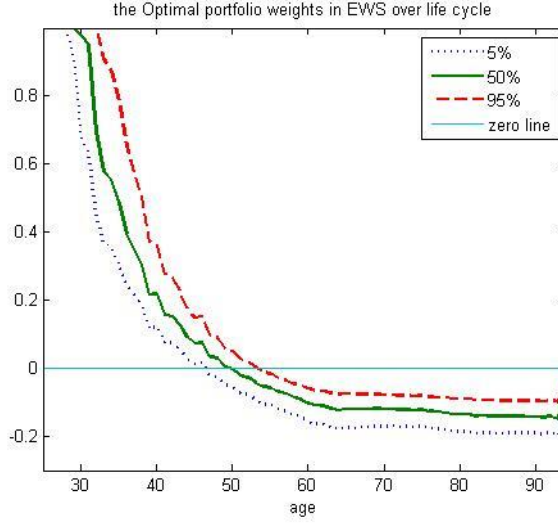


Figure 5: The 5%, 50% and 95% quantile distribution of the EWS portfolio weights (α_t^K) of entry cohorts over a lifetime in a setting with a constant asset default mix.

Next, we would like to check how optimal positions taken by individuals in the EWS change over the life cycle. This is shown in Figure 5, where we look for the entry 0 cohort in the quantiles of the portfolio weights in the EWSs. Note that the optimal weights start at 100% and gradually decrease to -17%, on average, in the retirement period. This cohort typically has a long position in the EWS until about age 50— and then a short position in the EWS after age 50. This is understandable, since the choice of EWS depends on available investable wealth and the remaining investment horizon.

Trading in EWSs over the life cycle has an important impact on investor wealth accumulation. Figure 6a compares the quantile distribution of the wealth accumulated with (A^{EWS}) and without a trading EWS (A^{noEWS}) for the entry cohort (age 25). For greater visibility, Figure 6b shows the relative difference of the two, $(A^{EWS} - A^{noEWS})/A^{noEWS}$. We make two outstanding observations. First, when equity performs well, the typical investor gets more upside (wealth increases by 15%) when young, and giving away some of the upside (wealth decreases by 10%) when old. However, when the equity market performs poorly, the older investor will get protection from the future young (wealth increases by 10–15%). Second, the median wealth level is 2–5% higher with trading in EWSs than without. The entry cohort thus has, on average, more wealth available for consumption, benefiting from improved risk exposure over the life cycle, the equity risk premium, and less-restrictive liquidity constraints (or borrowing constraints).

The ultimate motivation for trading in the swap market is to improve one's consumption profile over the life cycle. Figures 6c and 6d report on the distribution of the consumption paths for the now 25-year-old cohort with and without the possibility of being engaged in the swap market. We observe that during the accumulation phase, the new consumption profile becomes higher for all quantiles. This higher consumption can also be observed in the first years of the decumulation phase. Later in retirement, the median of the new profile is close to that of the old situation. However, one may observe a smaller spread around the median: especially the worst-case consumption (5% quantile) is better in the new situation. These changes in the consumption profiles are due to the re-allocation of equity and wage-growth risk over the life cycle via the EWS trading. The higher consumption and lower spreading are also observed for the other cohorts.

Figures 7a and 7b show the impact of EWS trading on wealth accumulation for the 60-year-olds at time 0. Figure 7a compares the quantile distribution of the wealth accumulated with and without trading EWSs for the cohort of age 60 at time 0. Figure 7b shows the relative difference between the two. We see that, when EWS is available, the investor has more protection (up to 10%) when equity performs poorly, but gives away some of the upside when equity performs well. On average, the current elderly have less wealth (up to 5%) accumulated due to their position in the EWS trade.

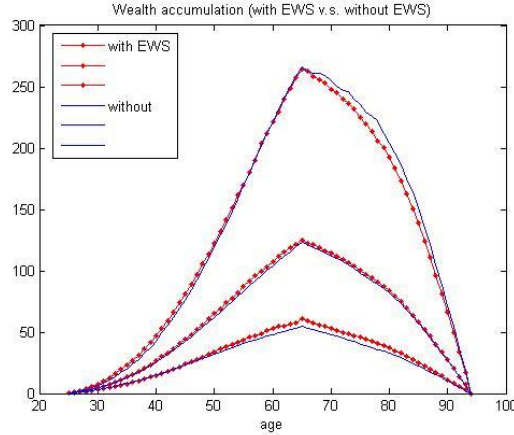


Figure 6a: The 5%, 50% and 95% quantile distribution of wealth accumulated with (A^{EWS}) and without EWS trading (A^{noEWS}) for the entry cohort (age 25).

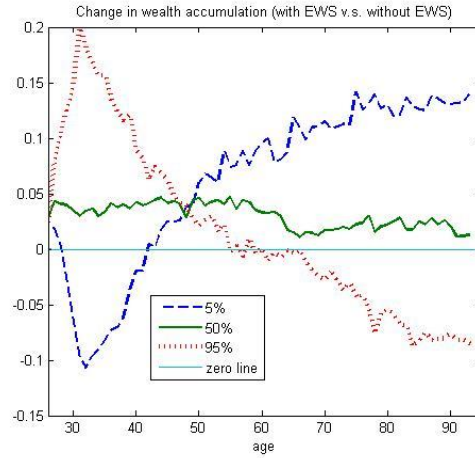


Figure 6b: The relative difference between the quantile distributions of wealth accumulated with and without EWS trading, $(A^{EWS} - A^{noEWS})/A^{noEWS}$, for the entry cohort (age 25).

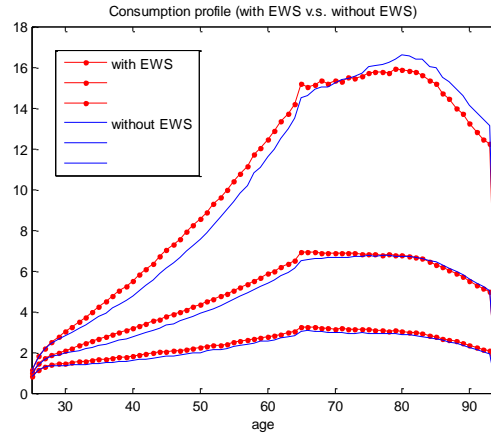


Figure 6c: The 5%, 50% and 95% quantile distribution of consumption with (C^{EWS}) and without EWS trading (C^{noEWS}) for the entry cohort (age 25).

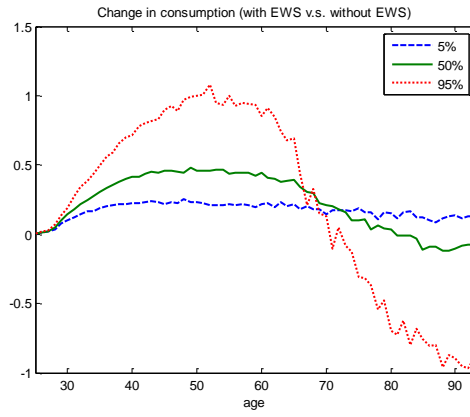


Figure 6d: The difference between the quantile distributions of wealth accumulated with and without EWS trading, ($C^{EWS} - C^{noEWS}$), for the entry cohort (age 25).

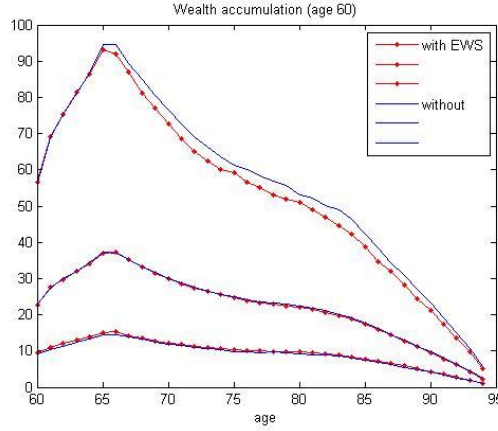


Figure 7a: The 5%, 50% and 95% quantile distribution of wealth accumulated with and without EWS trading for the initial 60-year-old cohort.

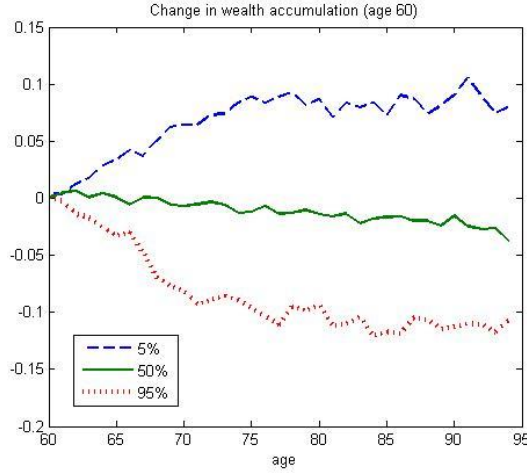


Figure 7b: The relative difference between the quantile distributions of wealth accumulated with and without EWS trading, $(A^{EWS} - A^{noEWS})/A^{noEWS}$, for the initial 60-year-old cohort.

The main results of the baseline variant are summarized in Table 1 in the row for the baseline case. We start by reporting the risk-aversion parameter γ and the default asset mix α_t^{DC} , consisting of 44% equity for all ages. We then document the main results regarding the median market size of the EWS market as a percentage of total wealth and the median premium m , both after 70 years. The α_{85}^K column reflects the median choice of an 85-year-old regarding his or her position in the EWS. The term α_{85}^K is equal to -12%, indicating that this person is short in stocks and long in wage growth. Figure 5 provides full information on the distribution of the preferred positions in the market by all cohorts. Finally, we show for this case the difference in expected payoffs between the two legs of the EWS swap, $E[R^E]$ and $E[R^G] + m$.

	γ	α^{DC}	Market size	α_{85}^K	m	$E[R^E] - (E[R^G] + m_t)$
Baseline	3	44%	6.5%	-12%	2.4%	1.8%

Table 1: Summary observations of the base case.

3.2 Impact of changes in the default asset mix

We consider two alternatives for the constant default mix, 60% and 27% equity, respectively, instead of the 44% used in the baseline case. In line with what one might expect, the step toward a riskier asset mix will lead to a lower m , increasing the payoff for those with a long position in the swap (i.e., the young) and decreasing for those with a short position (i.e., the elderly). This can be explained as follows.

Individuals can adjust their exposure to equity and wage growth only by changing their positions in the swap contract. Adjustment of the default asset mix from 44% to the risky 60% mix implies that the elderly are farther away from their preferred stake in equities. They are more willing to accept a lower expected return from the swap (i.e., $(E[R^G] + m_t) - E[R^E]$), in order to move toward their preferred position. This will imply a market process toward a lower m , all else being equal. Younger cohorts, on the other hand, are less willing to sell wage-growth risk in exchange for equity-risk exposure, since the default asset mix is closer to their preferred one. So they require a higher net return in taking over equity-risk exposure (i.e., $E[R^E] - (E[R^G] + m_t)$), which also leads to a market adjustment process to a lower m . Thus, the reset of the default asset mix to a riskier one will find a new equilibrium at a lower m . The expected payoff of the swap increases from 1.8% to 2.5%. The process toward a new equilibrium leads to higher activity in the swap market, since the market size ultimately increases from 6.5% to 8%. The variable α_{85}^K column, the average EWS position of the 85-year-old, is higher in absolute terms than in the baseline case. The elderly need a larger stake in the swap market to offset the increased default exposure in equities, with a larger negative stake in the EWSs.

When the default equity position is reduced to 27%, the opposite results are obtained. Now the elderly are closer to their preferred position, which reduces their willingness to trade, whereas the young are more willing to pay a higher m to obtain more equity exposure. So both the supply and demand sides push for the same outcome of a higher m and a lower relative market size.

	γ	α^{DC}	Market size	α_{85}^K	m	$E[R^E] - (E[R^G] + m_t)$
Baseline	3	44%	6.5%	-12%	2.4%	1.8%
More equity	3	60%	8.0%	-18%	1.7%	2.5%
Less equity	3	27%	4.2%	-8%	3.2%	1.0%

Table 2: Summary observations from comparing the base case with a changing default asset mix (more and less equity, respectively) for DC scheme with constant asset mix as default.

The finance literature recommends structuring the default asset mix according to the life cycle approach. This paper also incorporates the life cycle default-asset mix, and assesses its impact on the EWS market. We evaluate the demand, supply and resulting market-clearing prices of EWSs for participants in a DC pension plan with a life cycle (LC) default mix starting at 100% at age 25 and gradually declining to 44%

by age 65. Our purpose is to show the flexibility of the EWS construction, which is able to adapt to any given default setting.⁹

	γ	α_{65}^{DC}	Market size	α_{85}^K	m	$E[R^E] - (E[R^G] + m_t)$
Baseline	3	44%	6.5%	-12%	2.4%	1.8%
Life cycle	3	44%	4.0%	-6%	2.0%	2.2%

Table 3: Summary observations from comparing the base case with an alternative DC scheme with life cycle asset mix as default.

In comparison with the baseline, the initial asset mix for the younger cohorts is closer to their preference, so their willingness to be engaged in the EWS market is lower (and hence they require higher returns to attract them to participate in the EWS market, which contributes to a lower m). The resulting market size is also lower. Figure 9 shows that the equilibrium EWS premium follows the same course as for the baseline case (Figure 4), but at a lower level.

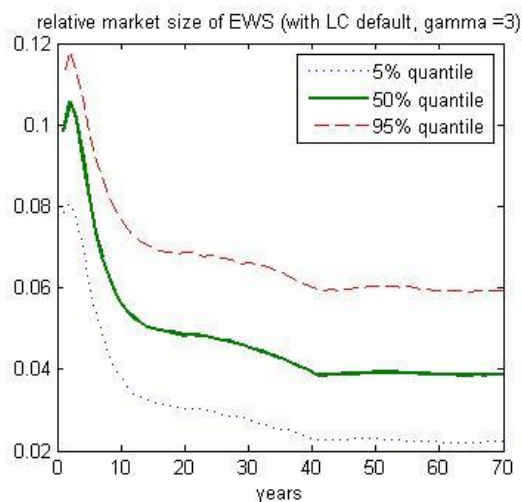


Figure 8: The 5%, 50% and 95% quantiles of the EWS market size relative to the total accumulated wealth of all coexisting cohorts over time in a setting with a life cycle (LC) default asset mix.

⁹ Even under an optimal individual DC scheme in the current market setting, there is still a need for EWSs because of market incompleteness and liquidity constraints (especially affecting the young).

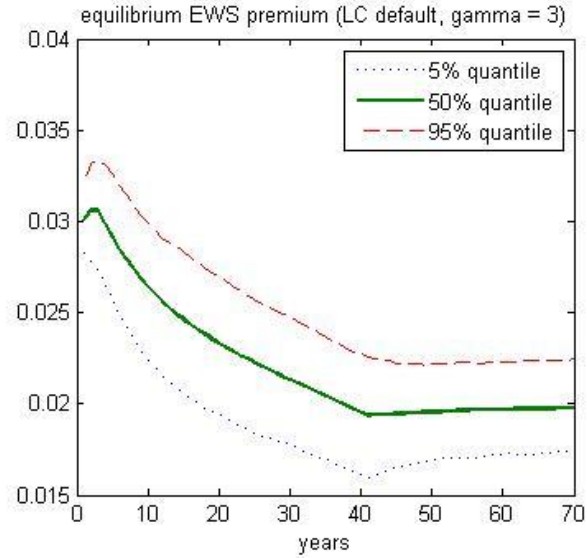


Figure 9: The 5%, 50% and 95% quantiles of the market-clearing premium (%) over the simulated 70-year horizon in a setting with a life cycle (LC) default-asset mix for a DC plan.

Following the entry cohort's choices over time, Figure 10 shows the chosen life cycle default exposure to equity with the blue line and the quantile distribution of their EWS position over a life cycle. The figure shows that younger cohorts start with a large positive position in EWSs, which then quickly reduces to 0% around middle age and then becomes negative.

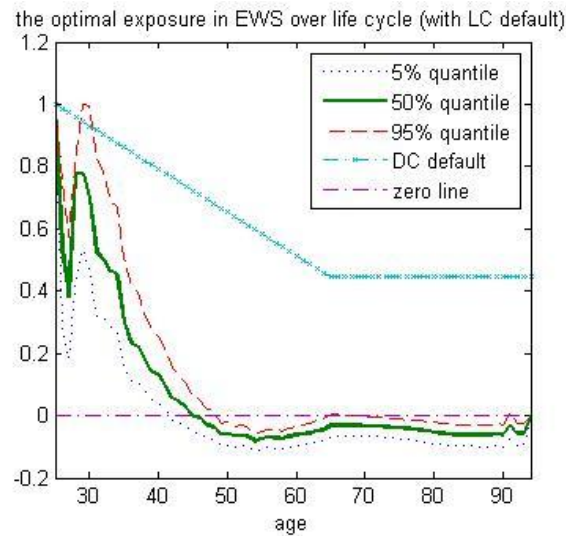


Figure 10: The optimal EWS position chosen by the entry cohort over a full life cycle. The x-axis displays their age and the y-axis shows exposure to equity from both the life cycle default and the EWS.

3.3 Risk aversion

Table 4 reports the impact of a higher risk-aversion parameter: specifically, an increase from three to five. We consider this sensitivity analysis for three cases: the first two cases for a fixed default mix of 27% equity, but with different risk-aversion parameters (three and five, respectively), and the third case for a life cycle mix with a share of 100% equity at age 25, linearly declining to 27% by age 65 and remaining constant during retirement.

When risk aversion increases, the elderly are less willing to accept equity risk while the relative attractiveness of wage growth risk increases. The elderly are thus more inclined to exchange equity risk for wage-growth risk, and will therefore accept a lower m to get rid of equity risk. The fall in m is greater when the life cycle mix acts as the default mix instead of the fixed default of 27% equity. With the life cycle mix, younger participants are closer to the mix preferred by them and so will ask for higher compensation to take over equity risk from the elderly.

	γ	α_{65}^{DC}	Market size	α_{85}^K	m	$E[R^E] - (E[R^G] + m_t)$
In DC scheme with fixed default-asset mix						
Less equity	3	27%	4.2%	-8%	3.2%	1.0%
Higher risk aversion	5	27%	4.0%	-8%	2.4%	1.8%
In DC scheme with life cycle default-asset mix						
Life cycle	3	44%	4.0%	-6%	2.0%	2.2%
Life cycle	5	27%	3.0%	+1%	1.3%	2.9%

Table 4: Summary observations from comparing the impact of different risk aversion.

3.4 The impact of aging

The magnitude of the market-clearing EWS premium also depends on the demographic structure (the sizes of various cohorts) of the considered population. In this section, the demographic structure follows that of the Dutch population projected by the United Nations in 2010. Figure 11 shows the demographic structure in 2010 and 2060 (projected). The 2060 projection includes foreseen improvements in longevity. For comparison, the stable demographic structure used so far is added to this figure. In this analysis we assume a fixed mix as the default portfolio.

Aging implies that the relative proportion of elderly persons increases over time, whereas the relative proportion of the young decreases. This shift in demographic structure means that the relatively small younger cohorts will receive more attractive returns from the relatively large elderly cohorts through EWS trade. The downward pressure on the EWS premium and the market size after the 20-year horizon are shown in Table 5, which indeed shows a reduction in the EWS premium to 2.7% and a reduction in market size to 6.5%.

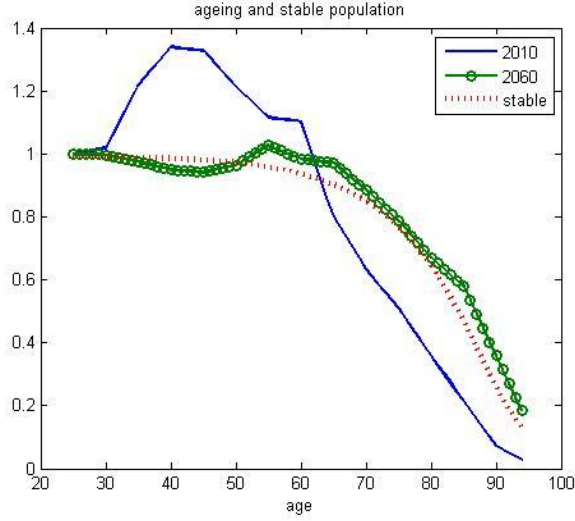


Figure 11: The (projected) demographic structure in 2010 and 2060 by the United Nations. The cohort size of 25-year-olds is normalized to one.

	γ	α^{DC}	Market size	α_{85}^K	m	$E[R^E] - (E[R^G] + m_t)$
Baseline	3	44%	7.5%	-12%	2.85%	1.4%
Aging	3	44%	6.5%	-12%	2.7%	1.5%

Table 5: Summary observations from comparing the base case to the case of aging for a 20-year horizon.

3.5. Impact of the correlation between equity returns and aggregate wage-growth rates.

When the equity returns and aggregate wage-growth rates are more positively correlated ($\rho = 0.4$) than the baseline assumption ($\rho = 0.2$), we observe a slightly lower market size and slightly less position-taking in the EWS by both younger and older cohorts, but no change in the EWS premium level in the long term. Because of the higher correlation between equity and wage-growth returns, all individuals (i.e., both the supply- and demand sides of the EWS) clearly reduce their exposure in EWSs without having a strong impact on the EWS premium. Table 6 summarizes the key comparisons.

	γ	α^{DC}	Market size	α_{85}^K	M	$E[R^E] - (E[R^G] + m_t)$
Baseline	3	44%	6.5%	-12%	2.4%	1.8%
Higher correlation	3	44%	5.9%	-10%	2.4%	1.8%

Table 6: Summary observations from comparing the base case to the higher-correlation case.

4. Conclusion

An important issue in pension plan design is the search for a more efficient way of allocating and trading risk across generations. We propose market-based voluntary risk trading among coexisting generations rather than mandatory risk-sharing rules. We find that the proposed EWS market can be established between young and old generations, for three reasons: First, the market helps to complete a missing market of wage-linked claims; second, it helps to lift the liquidity (borrowing) constraints of the young; and third, the EWS market offers individuals a flexible way to realize their preferred exposure to wage- and equity risk. This type of swap market is transparent, effective, flexible and fair in improving the welfare of both younger and older individuals. We have developed a model to price the EWS traded among 70 coexisting cohorts within realistic life cycle settings. We find that there is always a market for EWS trading among coexisting generations. The market-clearing premium for EWSs may vary, however, depending on multiple state variables (including the state of the economy, demographics and aggregated human and financial capital).

References

- Beetsma, R. and A. Buccioli (2011), Differentiating indexation in Dutch pension funds, *De Economist*, vol. 159, no. 3, 323–360.
- Benzoni, L., P. Collin-Dufresne and R.S. Goldstein (2007), Portfolio choice over the life-cycle when the stock and labor markets are cointegrated, *Journal of Finance*, vol. 62, 2123–2167.
- Blake, D. A. Cairns and K. Dowd (2008), Turning pension plans into pension planes: What investment strategy designers of defined-contribution pension plans can learn from commercial aircraft designers, City University Pensions Institute discussion paper, PI-806.
- Carroll, C.D. (2006), The method of endogenous gridpoints for solving dynamic stochastic optimization problems, *Economics Letters*, vol. 91, 312–320.
- Cocco, J.F., F.J. Gomes and P.J. Maenhout (2005), Consumption and portfolio choice over the life cycle, *Review of Financial Studies*, vol. 18, no. 2, 491–533
- Cui, J. (2009), What should your DC default be? SSRN working paper, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1343758
- Cui, J., F. de Jong and E. Ponds (2011), Intergenerational risk sharing within funded pension schemes, *Journal of Pension Economics and Finance*, vol. 10, no. 1, 1–29.
- De Jong, F. (2008a), Pension fund investments and the valuation of liabilities under conditional indexation, *Insurance: Mathematics & Economics*, vol. 42, no. 1, 1–13.
- De Jong, F. (2008b), Valuation of pension fund liabilities in incomplete markets, *Journal of Pension Economics and Finance*, vol. 7, no. 3, 277–294.
- Gomes, F. and A. Michaelides (2008), Asset pricing with limited risk sharing and heterogeneous agents, *Review of Financial Studies*, vol. 21, no. 1, 415–448.
- Merton, R. (2010), Solutions for the future, *Pensions Insight*.
- Molenaar, R. and E.H.M. Ponds (2012), Risk sharing and individual life cycle investing in funded collective pensions, *Journal of Risk*, vol. 15, no. 2, 103–124.
- Van Binsbergen, J., D. Broeders, F. de Jong and R. Koijen (2013), Collective pension schemes and individual choice, Netspar Design Paper 19, Netspar.
- Viceira, L.M. (2008), Life-cycle funds. In: Annamaria Lusardi, ed., *Overcoming the Saving Slump: How to Increase the Effectiveness of Financial Education and Saving Programs*, University of Chicago Press.

Appendix: Solution method

A.1 Rewriting the optimization problem

Since optimization is from each individual's point of view, we first simplify the subscript (t, t_x, x) to t . Then we normalize the consumption and wealth variables by the permanent income Y_t . The normalized variables are denoted in small letters throughout; for example, $w_t = W_t/Y_t$. Then, we rewrite the normalized objective function in recursive form and then together with the normalized budget constraints as follows:

$$v(w_t) = \max \frac{c_t^{1-\gamma}}{1-\gamma} + \beta E_t \left[(R_{t+1}^G R_{t+1}^N)^{1-\gamma} v(w_{t+1}) \right]$$

such that

$$w_{t+1} = (w_t - c_t) [\alpha_t R_{t+1}^E + (1 - \alpha_t) R^f + \alpha_t^K (R_{t+1}^E - R_{t+1}^G - m_t)] + (1 - \tau^y) R_{t+1}^u$$

$$w_t - c_t > 0$$

$$-1 < \alpha_t^K < 1$$

with $\alpha_t^K = \frac{K_t}{w_t - c_t}$.

The first-order conditions with respect to α_t^K and c_t are:

$$\begin{aligned} 0 &= \beta E_t \left[v'(w_{t+1}) (R_{t+1}^E - R_{t+1}^G - m_t) (R_{t+1}^G)^{-\gamma} \right] \\ u'(c_t) &= \beta E_t \left[v'(w_{t+1}) (\alpha_t R_{t+1}^E + (1 - \alpha_t) R^f + \alpha_t^K (R_{t+1}^E - R_{t+1}^G - m_t)) (R_{t+1}^G)^{-\gamma} \right] \end{aligned}$$

The envelope theorem implies that $u'(c_t) = v'(w_t)$. Replacing $v'(w_{t+1})$ by $u'(c_{t+1})$ in the first-order conditions, we obtain:

$$\begin{aligned} 0 &= \beta E_t \left[u'(c_{t+1}^*[w_{t+1}]) (R_{t+1}^E - R_{t+1}^G - m_t) (R_{t+1}^G)^{-\gamma} \right] \\ u'(c_t) &= \beta E_t \left[u'(c_{t+1}^*[w_{t+1}]) (\alpha_t R_{t+1}^E + (1 - \alpha_t) R^f + \alpha_t^K (R_{t+1}^E - R_{t+1}^G - m_t)) (R_{t+1}^G)^{-\gamma} \right] \end{aligned}$$

where $c_{t+1}^*[w_{t+1}]$ denotes the optimal consumption policy at time $t + 1$, which is already obtained during the dynamic program procedure.

Using first-order conditions with respect to α_t^K , we can pin down the optimal portfolio policy, denoted $(\alpha_t^{K*} | \{(w_t - c_t)^{(i)}, m_t^{(j)}\})$, namely, the optimal choice of α_t^{K*} for any given investable wealth $(w_t - c_t)^{(i)}$ and any given EWS premium $m_t^{(j)}$. To implement the numerical grid search, $(w_t - c_t)^{(i)}$ denotes the i th discrete value in a predefined set of grid points for investable wealth and $m_t^{(j)}$ denotes the j th discrete value in a predefined set of grid points for the EWS premium.

A.2 Optimization

The numerical implementation of the first-order conditions is inspired by the endogenous gridpoint method explained in Carroll, C.D. (2006). Starting from the final period, the optimal consumption policy is to consume all the remaining wealth, that is, $c = w$. This yields the terminal condition for the backward induction procedure:

- Step one: Construct a two-dimensional grid for investable wealth a and the EWS premium m . For any given values of $\{a, m\}$, determine the optimal portfolio choice $(\alpha_t^{K*} | \{(w_t - c_t)^{(i)}, m_t^{(j)}\})$ for each investable wealth a and the EWS premium m using the first-order condition as explained in A.1.
- Step two: Using the optimal portfolio choice, simulate the portfolio return and subsequently simulate the cash-on-hand x of the next period for the given investable wealth a and the EWS premium m .
- Step three: Calculate the optimal consumption policy $c^*(a, m)$ for the given investable wealth a and the EWS premium m using the expression for $u'(c_t)$ as derived in A.1.
- Step four: Determine endogenously the cash-on-hand of this current period, where $x = a + c^*(a, m)$.

A.3 Simulation and market clearing

Simulate the behavior of 70 co-existing age cohorts over 70 years. For each time period, simulate the individuals' behavior for every possible EWS premium m . Then, aggregate the EWS supply and demand of the coexisting cohorts to determine the market-clearing EWS premium:

$$\sum_i a_t^{(i)} \alpha_t^{*K, (i)} = 0$$

where i denotes all the individuals from all co-existing cohorts presented in this market.